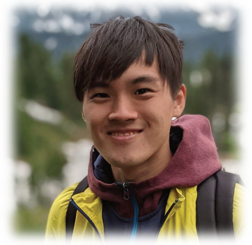
Stability Analysis of Quadratic Systems using Quadratic Constraints



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BACKGROUND:

Nonlinear quadratic dynamics arise in models of fluid flows, populations, and epidemics. For the purpose of system design and control, certifiable stability is often desired in order to guarantee the system behaves as intended.

METHODS:

- Estimate region of attraction (ROA) of a stable equilibrium by absolute stability using Quadratic Constraints (QCs).
- A QC is an abstraction of the nonlinearity ϕ as a quadratic inequality of input and output signals (x and z in the Lur'e system in the flow chart on the right).

$$\begin{bmatrix} x \\ z \end{bmatrix}^\top M \begin{bmatrix} x \\ z \end{bmatrix} \geq 0,\ z = \phi(x), \forall x \in \mathcal{E}_\alpha.$$

- A new class of QC (Valley QC) is proposed to bound the behavior along zero directions of the nonlinearity, e.g., $x_1 = 0$ or $x_2 = 0$ for $\phi(x) = x_1 x_2$.
- Semidefinite programs (SDP) are formulated to establish Lyapunov stability certificates and ROA estimates.

RESULTS

A 2-state numerical example is presented. The figure shows the phase portraits, unstable region (yellow), and ROA estimates with (red) and without (blue) newly proposed QCs.

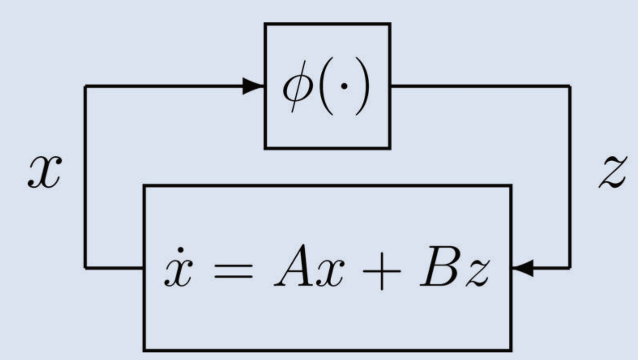
FUTURE DIRECTION

- Computational efficiency: proposed analysis solves SDPs over a grid of local region size. Methods in parametric optimization might be able to reduce the complexity.
- QCs for polynomial: the proposed Valley QC can be extended to higher-order polynomials, and thus enable absolute stability analysis for a broader class of systems.
- Data-driven QCs: there are opportunities to construct QCs from sampled input-output data of nonlinearities. Sample complexity results from scenario optimization can potentially provide statistical guarantees.

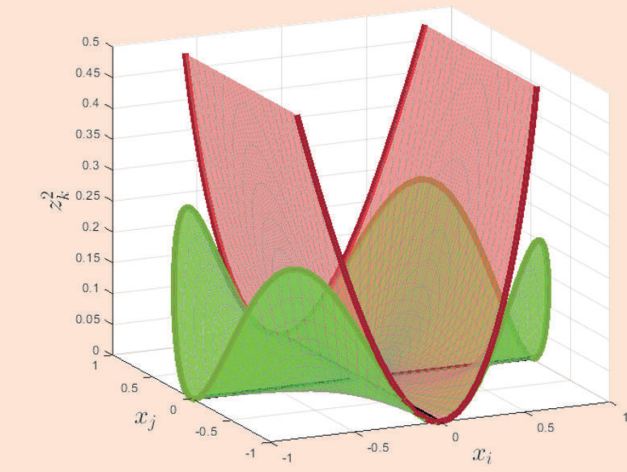
Improved *region of attraction* estimation for *nonlinear quadratic dynamics* using **Valley Quadratic Constraints that bound zero directions.**

Separating Nonlinearity → Characterizing Nonlinearity → Formulating Local Stability Condition → Finding Largest Spherical ROA

Lur'e System



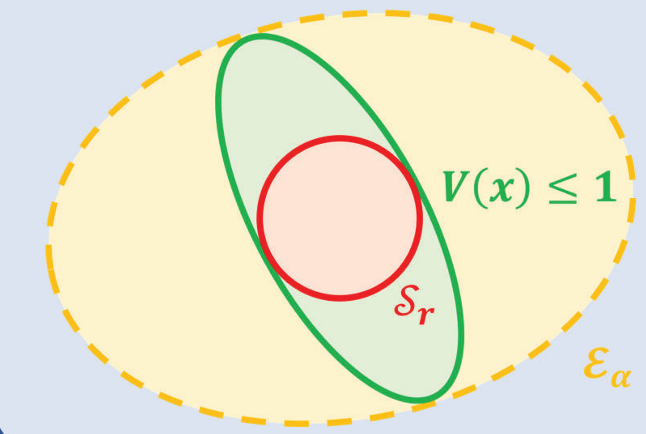
Quadratic Constraints

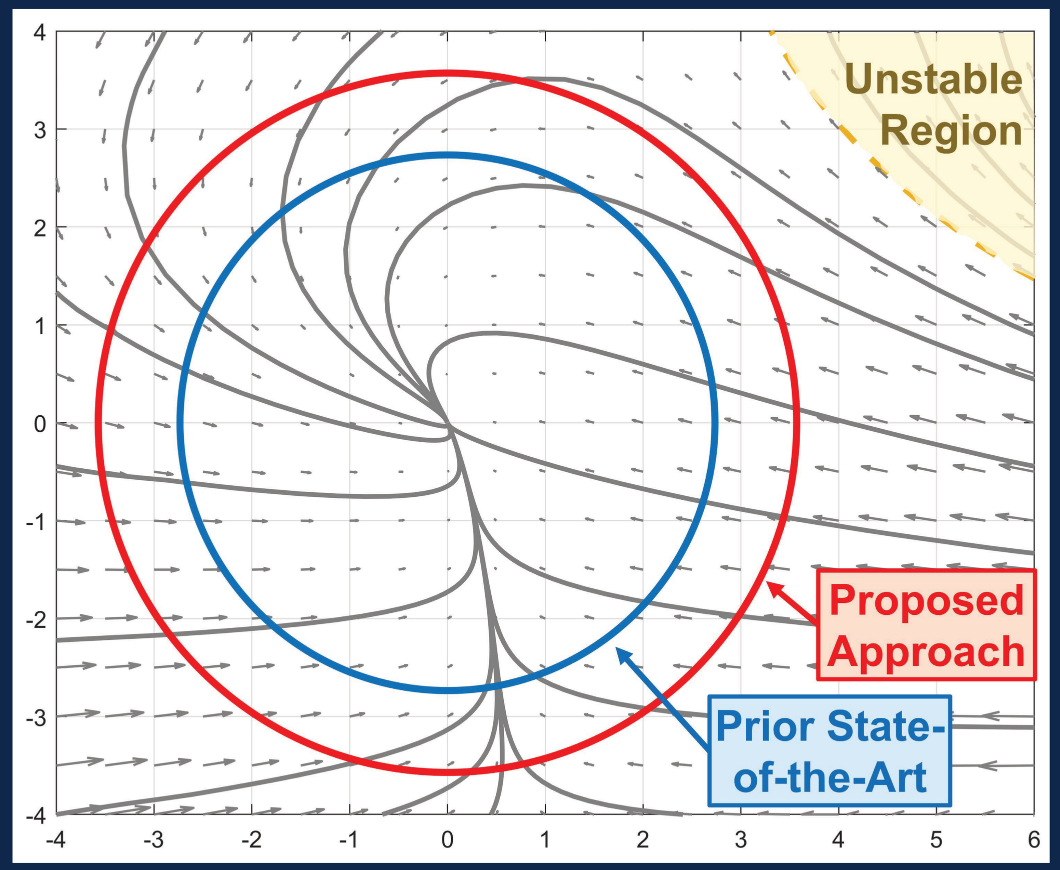


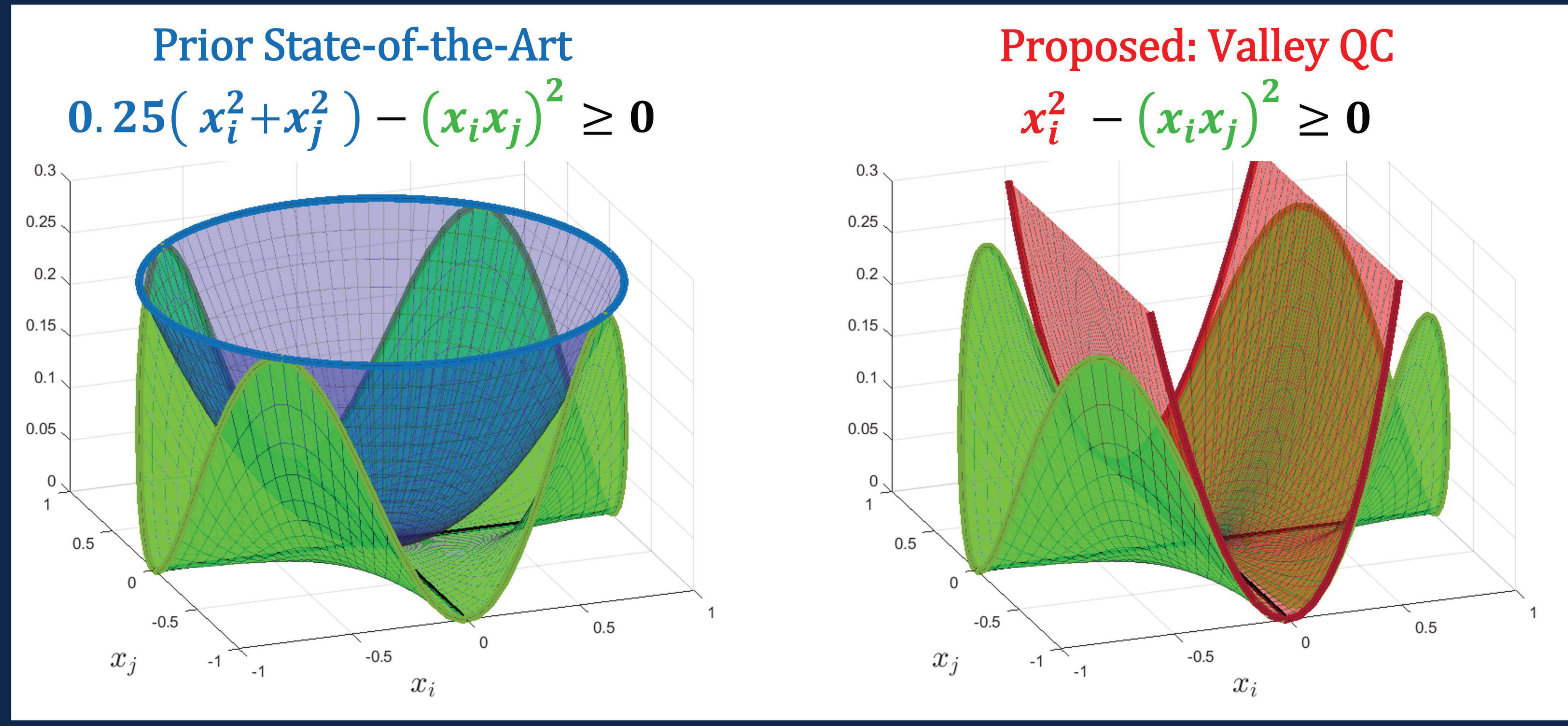
S-Procedure

$$\dot{V}(x) < 0$$
$$\forall x \in \mathcal{E}_\alpha$$

Semidefinite Program







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