



**MICHIGAN
ENGINEERING**
UNIVERSITY OF MICHIGAN

Safety-Critical Control with Sector-Bounded Uncertainties using Robust Control Barrier Function

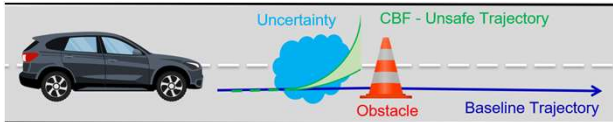
Shih-Chi Liao¹, Jyot Buch², Peter Seiler¹

¹Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI.

²Aerospace Engineering and Mechanics, University of Minnesota, Twin Cities, MN.



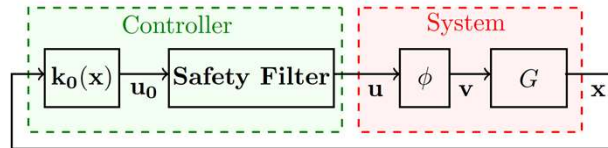
Motivation



- Safety has highest priority for many autonomous systems.
- Control Barrier Function (CBF) [1] is a method to ensure safety. However, CBF requires perfect model of the system.
- Robust CBF (RCBF) is needed to handle uncertainties.

Objectives

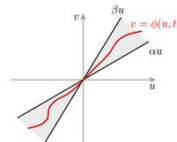
Design a safety filter that minimally alters baseline controller and satisfy safety requirements despite the model uncertainty ϕ .



Uncertainty and RCBF

Sector-bounded Uncertainty $\phi(u)$:

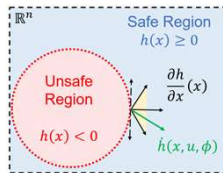
- A function with uncertain output
- Bound by two linear functions
- Can rewrite into norm-bounded form with an additive uncertain input w and a magnitude parameter θ



Robust Control Barrier Function $h(x)$:

- A function describes safety
- System is safe for all uncertainty in ϕ if the condition holds:

$$\inf_{w \in \mathcal{W}} \dot{h}(x, u, w) + \eta(h(x)) \geq 0$$



Robust Safety Filter

Safety filter is cast as an optimization problem with RCBF constraint:

$$u^*(x) = \arg \min_u \|u - u_0\|_2$$

$$\text{s.t.} \quad \inf_{w \in \mathcal{W}} \dot{h}(x, u, w) + \eta(h(x)) \geq 0$$

- Can be reformulated as Second-Order Cone Program (SOCP), a convex problem that can be solved efficiently
- If $u^*(x)$ is feasible, $u^*(x)$ render the system safe forward in time.

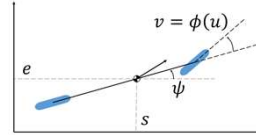
Vehicle Lateral Control

Obstacle avoidance scenario:

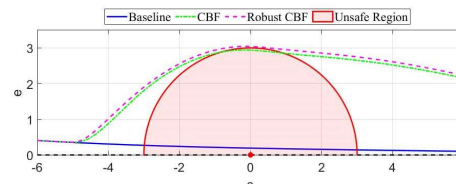
- Constant vehicle forward speed
- Uncertainty in turning interaction

Vehicle model:

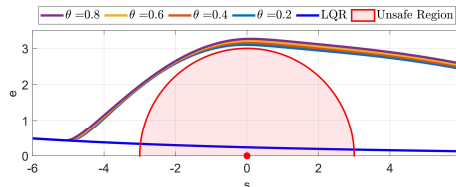
- e : lateral deviation from path
- s : path progression
- ψ : vehicle heading
- u : steering input



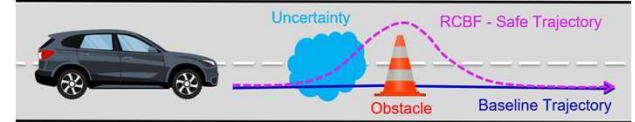
Simulation with the worst-case uncertain system:



Simulation with varying robustness level of RCBF design:



Conclusions



With the RCBF-based safety filter, the autonomous system can:

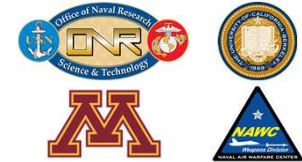
- Provide robust safety guarantee
- Consider uncertainties explicitly
- Enable applications for a more general setting

In our paper [2], we further discuss followings:

- Proof of robust safety guarantee
- Reformulation and derivation of SOCP
- Continuity of the RCBF-based safety filter
- Limitation of RCBF-based safety filter
- Extensions and future work

Acknowledgements

- UC Berkeley:
- Prof. Murat Arcaç
 - Kata Schweidel
 - Adnane Saoud
- NAWCWD China Lake:
- Doug Philbrick



References

- [1] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," IEEE Transactions on Automatic Control (TAC), 2016
- [2] J. Buch, S. Liao, and P. Seiler, "Robust control barrier functions with sector-bounded uncertainties," arXiv:2109.02537, 2021.

Our paper:



Our code:

